1 SOAC OBSERVATION DATA FILE (SOOBDF)

(1) GENERAL

The SOOBDF is created for the purpose to save tracking data in SELENE Observation Data Archive Center (SOAC). The SOOBDF consists of SOAC Header and OBDF.

(2) FILE SPECIFICATION

The SOOBDF is described in Table 1-1.

Table 1-1. SOOBDF specification

Item	Content
File Identification	SOAC Observation Data File (SOOBDF)
File Type	Files created for each spacecraft/station/data type
File Format	Text file (Character Code = ASCII)
Record Length	Variable length SOAC Header Record: 129 byte fixed TACC-Normalized Observation Data: Equivalent to OBDF
Maximum Number of Records	N/A
Data	SOAC Header and TACC-Normalized Observation Data
File Creator	SELENE Information Sub-System / SOAC Data Conversion Program
Note	

(3) FILE STRUCTURE

Table 1-2. SOOBDF structure

Block	Record	Item	Detailed Description
SOAC Header	SOAC Header Record	File Classification, Creation Date and Time, Data Block Length, Spacecraft ID, Spacecraft Name, Storage Start/End Date and Time, Station Name, Data Type Name	Table 1-3
TACC-normalized Observation Data	Equivalent to OBDF. See "2 Observation Data File" for detailed description.		–

(4) RECORD

Detailed Descriptions of Records in the SOOBDF are provided in Table 1-3.

[Convention]

'␣': Space

'9999': Number with zero padding (ex., '0001'). 'ZZZ9': Number without zero padding (ex., '␣␣␣1').

'S': Sign (positive: '+', negative: '-'). 's': Sign (positive:space, negative: '-').

1PE22.15 form: 's9.999999999999999ES99' form. (To maintain 16-digit precision, integer in the base shall not be zero.)

Ex., '1.234567890123456E-02' shall not be expressed as '0.123456789012346E-01'.

Table 1-3. SOAC Header Record

File	SOOBDF	SOAC Header Record			
Byte	Item Name	Type	Length	Description	Note
0	SOAC Header ID	C	8	‘#!Head:␣’ fixed	
8		C	1	Blank	
9	File Classification	C	8	‘SOOBDF␣␣’ fixed	
17		C	1	Blank	
18	File Creation Date	C	10	Machine date when the file is created ‘YYYY-MM-DD’ format	(UTC)
28		C	1	Blank	
29	File Creation Time	C	8	Machine time when the file is created ‘hh:mm:ss’ format	(UTC)
37		C	1	Blank	
38	Data Block Length	C	12	File size without SOAC Header ‘ZZZZZZZZZZZ9’ format	(byte)
50		C	1	Blank	
51	Spacecraft ID	C	2	‘34’: Main ‘35’: Rstar ‘36’: Vstar	
53		C	1	Blank	
54	Spacecraft Name	C	16	‘SELENE-M␣␣␣␣␣␣␣␣’: Main ‘SELENE-R␣␣␣␣␣␣␣␣’: Rstar ‘SELENE-V␣␣␣␣␣␣␣␣’: VstarS	
70		C	1	Blank	
71	Storage Start Date	C	10	‘YYYY-MM-DD’ format	(UTC)
81		C	1	Blank	
82	Storage Start Time	C	8	‘hh:mm:ss’ format	(UTC)
90		C	1	Blank	
91	Storage End Date	C	10	‘YYYY-MM-DD’ format	(UTC)
101		C	1	Blank	
102	Storage End Time	C	8	‘hh:mm:ss’ format	(UTC)
110		C	1	Blank	
111	Station Name	C	8	‘OKN1␣␣␣␣’: Okinawa Tracking and Communication Station ‘KTU1␣␣␣␣’: Katsuura Tracking and Communication Station ‘MSD1␣␣␣␣’: Masuda Tracking and Communication Station ‘SNT1␣␣␣␣’: Santiago Tracking and Communication Station ‘PRT1␣␣␣␣’: Perth Tracking and Communication Station ‘MSP1␣␣␣␣’: Maspalomas Tracking and Communication Station ‘KRN1␣␣␣␣’: Kiruna Tracking and Communication Station ‘KSC34␣␣␣’: Uchinoura Space Center ‘UDSC64␣␣’: Usuda Deep Space Center	
119		C	1	Blank	
120	Data Type Name	C	8	‘RA2␣␣␣␣␣’: 2-Way Range ‘DP2␣␣␣␣␣’: 2-Way Doppler ‘SDP4␣␣␣␣’: 4-Way Doppler (Rstar)	
128	Line Feed Code	C	1	0x0A	

(5) Appendix A
N/A

2 OBSERVATION DATA FILE (OBDF)

(1) GENERAL

The OBDF contains engineering data of range and Doppler measurements.

(2) FILE SPECIFICATION

The OBDF is described in Table 2-1.

Table 2-1. OBDF specification

Item	Content
File Identification	Observation Data File (OBDF)
File Type	Files created for each spacecraft/station/data type
File Format	Text file (Character Code = ASCII)
Record Length	Variable length (End of each record is Line Feed Code.) File Control Information Record: 25 byte, 36 byte Pass Control Information Record: 37 byte, (37 byte), 29 byte, 31 byte, 25 byte, 22 byte, 22 byte, 44 byte, 44 byte Preprocessing Information Record: 43 byte, 43 byte, 27 byte, 27 byte, 44 byte, 26 byte Observation Data Information Record: 92 byte
Maximum Number of Records	N/A
Data	Engineering data of range and Doppler measurements
File Creator	Orbit Determination Sub-System / Format Conversion Program from data obtained by ISAS ground-stations Orbit Determination Sub-System / Format Conversion Program from data obtained by GN ground-stations
Note	

(3) FILE STRUCTURE

Table 2-2. OBDF structure

Block	Record	Item	Detailed Description
File Control Block	File Control Information Record	File Classification, File Creation Date and Time	Table 2-3
Pass Control Information	Pass Control Information Record	Spacecraft Name,Station Name, Pass ID, Data Type, Uplink Band, Downlink Band, Reference Frequency, Station Delay	Table 2-4
Preprocessing Information	Preprocessing Information Record	Data Start/End Time, Number of Stored Data, Number of Rejected Data, modulo-M, Count Interval	Table 2-5
Observation Data Information	Observation Data Information Record #1	Time Tag, Observation Data, Azimuth Angle at Observation, Elevation Angle at Observation, Temperature at Observation, Relative Humidity at Observation, Pressure at Observation	Table 2-6
	⁞	(Repeat of Observation Data Information for the number of data)	
	Observation Data Information Record #n	Time Tag, Observation Data, Azimuth Angle at Observation, Elevation Angle at Observation, Temperature at Observation, Relative Humidity at Observation, Pressure at Observation	–

(4) RECORD

Detailed Descriptions of Records in the OBDF are provided in Tables 2-3, 2-4, 2-5, and 2-6.

[Convention]
'␣': Space
'9999': Number with zero padding (ex., '0001'). 'ZZZ9': Number without zero padding (ex., '␣␣␣1').
'S': Sign (positive: '+', negative: '-'). 's': Sign (positive:space, negative: '-').
1PE22.15 form: 's9.999999999999999ES99' form. (To maintain 16-digit precision, integer in the base shall not be zero.)
Ex., '1.234567890123456E-02' shall not be expressed as '0.123456789012346E-01'.

Table 2-3. File Control Information Record

File	OBDF	File Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	File Classification Label	C	20	'file_name␣␣␣␣␣␣␣␣␣␣=' format	
20	File Classification	C	4	'OBDF' fixed	
24	Line Feed Code	C	1	0x0A	

File	OBDF	File Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	File Creation Date and Time Label	C	20	'file_create␣␣␣␣␣␣␣␣=' format	
20	File Creation Date and Time	C	15	'yyyymmdd␣hhmmss' format	
35	Line Feed Code	C	1	0x0A	

Table 2-4. Pass Control Information Record

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Spacecraft Name Label	C	20	‘spacecraft_name␣␣␣␣=’ format	
20	Spacecraft Name	C	16	‘SELENE-M’ ‘SELENE-R’ Note 1) ‘SELENE-V’	Left justification
36	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	2nd Spacecraft Name Label	C	20	‘spacecraft_name_2nd=’ format	
20	2nd Spacecraft Name	C	16	‘SELENE-M’ Note 2)	Left justification
36	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Station Name Label	C	20	‘station_name␣␣␣␣␣␣␣=’ format	
20	Station Name	C	8	‘OKN1␣␣␣␣’: Okinawa Tracking and Communication Station ‘KTU1␣␣␣␣’: Katsuura Tracking and Communication Station ‘MSD1␣␣␣␣’: Masuda Tracking and Communication Station ‘SNT1␣␣␣␣’: Santiago Tracking and Communication Station ‘PRT1␣␣␣␣’: Perth Tracking and Communication Station ‘MSP1␣␣␣␣’: Maspalomas Tracking and Communication Station ‘KRN1␣␣␣␣’: Kiruna Tracking and Communication Station ‘KSC34␣␣␣’: Uchinoura Space Center ‘UDSC64␣␣’: Usuda Deep Space Center	Left justification
28	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Pass ID Label	C	20	‘pass_id␣␣␣␣␣␣␣␣␣␣␣␣=’ format	
20	Pass ID	C	10	‘yymmddnnmm’ format nn: Visible pass number of the day mm: Division number (Default = 00)	
30	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Data Type Label	C	20	‘data_type_name␣␣␣␣␣=’ format	
20	Data Type	C	4	‘RA2␣␣␣␣␣’: 2-Way Range ‘DP2␣␣␣␣␣’: 2-Way Doppler ‘SDP4␣␣␣␣’: 4-Way Doppler (Rstar)	
24	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Uplink Band Label	C	20	‘uplink_band␣␣␣␣␣␣␣=’ format	
20	Uplink Band	C	1	‘S’: S band	
21	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Downlink Band Label	C	20	‘downlink_band␣␣␣␣␣=’ format	
20	Downlink Band	C	1	‘S’: S-band ‘X’: X-band	
21	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Reference Frequency Label	C	20	‘standard_freq␣␣␣␣␣=’ format	
20	Reference Frequency	C	23	-1.1234567890123456E+12	(Hz)
43	Line Feed Code	C	1	0x0A	

File	OBDF	Pass Control Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Station Delay Label	C	20	‘station_delay␣␣␣␣␣=’ format	
20	Station Delay	C	23	-1.1234567890123456E+12 Applicable if Data Type is RA2 and correction is required; otherwise 0.0.	(second)
43	Line Feed Code	C	1	0x0A	

Note 1) If Data Type is SDP4, this record is fixed to be SELENE-R.
Note 2) If Data Type is SDP4, this record is fixed to be SELENE-M. Otherwise this record is blank.

Table 2-5. Preprocessing Information Record

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Data Start Time Label	C	20	‘data_start␣␣␣␣␣␣␣␣␣=’ format	
20	Data Start Time	C	22	‘yyyymmdd␣hhmmss.ssssss’ format	
42	Line Feed Code	C	1	0x0A	

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Data End Time Label	C	20	‘data_end␣␣␣␣␣␣␣␣␣␣␣=’ format	
20	Data End Time	C	22	‘yyyymmdd␣hhmmss.ssssss’ format	
42	Line Feed Code	C	1	0x0A	

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Number of Stored Data Label	C	20	‘stored_data_no␣␣␣␣␣=’ format	
20	Number of Stored Data	C	6	‘123456’ format	
26	Line Feed Code	C	1	0x0A	

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Number of Rejected Data Label	C	20	‘rejected_data_no␣␣␣=’ format	
20	Number of Rejected Data	C	6	‘123456’ format Note 3)	
26	Line Feed Code	C	1	0x0A	

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	modulo-M Label	C	20	‘modulo_m␣␣␣␣␣␣␣␣␣␣␣=’ format	
20	modulo-M	C	23	‘-1.1234567890123456E+12’ format	
43	Line Feed Code	C	1	0x0A	

File	OBDF	Preprocessing Information Record			
Byte	Item Name	Type	Length	Description	Note
0	Count Interval Label	C	20	‘tc␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣=’ format	
20	Count Interval	C	5	‘12345’ format For Doppler data, count interval in seconds is multiplied by 100; otherwise '00000'.	
25	Line Feed Code	C	1	0x0A	

Note 3) Rejected data have already been removed from the file.

Table 2-6. Observation Data Information

File	OBDF	Observation Data Information			
Byte	Item Name	Type	Length	Description	Note
0	Time Tag	C	22	‘yyyymmdd␣hhmmss.ssssss’ format	(UTC)
22		C	1	Blank	
23	Observation Data	C	23	‘-1.1234567890123456E+12’ format	See Appendix B [Tracking Data Format]
46		C	1	Blank	
47	Azimuth Angle at Observation	C	8	’123.1234’ format	(degrees)
55		C	1	Blank	
56	Elevation Angle at Observation	C	7	’12.1234’ format	(degrees)
63		C	1	Blank	
64	Temperature at Observation	C	8	’-12.1234’ format	(°C)
72		C	1	Blank	
73	Relative Humidity at Observation	C	8	’123.1234’ format	(%)
81		C	1	Blank	
82	Pressure at Observation	C	9	’1234.1234’ format	(mb)
91	Line Feed Code	C	1	0x0A	

(5) Appendix B

[Tracking Data Format]

```
# UTC time tag.   data(sec,Hz,rad)        Az(deg), El(deg),temp(C), humid(%),press(mb)
#---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8----+----9----+
yyyymmdd hhmmss.ssssss -1.1234567890123456E+12 123.1234 12.1234 -12.1234 123.1234 1234.1234
                      |                     |  |              | |                         |
                      +----------+----------+  +------+-------+ +------------+------------+
                                 |                    |                      |
                        Observation Data         Angle Data,          Weather Data
            For 2-way range    (sec)      Az(deg), El(deg), temp(C), humid(%), press(mb)
            For 2-way Doppler   (Hz)      Az(deg), El(deg), temp(C), humid(%), press(mb)
            For 4-way Doppler   (Hz)      Az(deg), El(deg), temp(C), humid(%), press(mb)
```

*) Data Block length is fixed to be 92 byte for 1 record. The last byte has to be Line Feed (LF).

**) The time-tag, *tt*, on the data file refers to the mid-interval value of the count interval *Tc* of the data file. If *ta* is the start and *tb* the end of the count interval, then

$$tb - ta = Tc$$

and

$$tt = (tb + ta) / 2 \quad .$$

The observation value as given on the file is computed from Doppler count values *ID* as

$$(ID(tb) - ID(ta)) / Tc$$

***) Weather data are listed, but are not measured every second. And they are available only for UDSC64 data.

[2-way and 4-way Data Conversion]

For processing SELENE tracking data, for example, further averaging 2-way data or converting from frequency into range rate, ancillary information about onboard instruments and transmitter at ground station is required. See *Iwata et al.* (2001) and *Namiki et al.* (2010) for details of instrumental design and estimated accuracies of the observation. As for computational algorithm, see an attached document by *Goossens*.

When using the four-way data, the following should be taken into account. There is a bias frequency involved:

bias frequency= F_{REF} - ($K2 + K5 * K6$) * f_{sent},

with F_{REF} = 8456.125 MHz. And with this, the full observable becomes

4-way range rate = 0.5 * (average frequency - bias frequency) * c / ($K5 * K6 * f_{sent}$)

Note that the factor 0.5 is included to make the distinction between round-trip measurements, and "one leg" measurements. See the attached document by *Goossens* for turn-around ratios, *K2*, *K5*, and *K6*. Reference frequency, f_{sent}, is given in the SOOBDF (Table 2-4).

Goossens, S., Doppler data processing for orbit determination, 2011.

Iwata, T., M. Takahashi, N. Namiki, H. Hanada, N. Kawano, K., Heki, K. Matsumoto, and T. Takano, Mission Instruments for Lunar Gravity Measurements using SELENE Sub- satellites, *J. Geod. Soc. Japan.*, *47*, 558-563, 2001.

Namiki, N., T. Iwata , N. Kawano, F. Fuke, N. Tateno, K. Asari, H. Noda, Y. Kono, H. Hanada, Y. Yahagi, Z. Yamamoto, K. Tanaka, M. Yamada, K. Matsumoto, S. Goossens, Ground compatibility tests for gravity measurement of SELENE: Accuracies of two- and four-way Doppler and range measurements, *Space Sci. Rev.*, *154*, 103–121, doi:10.1007/s11214-010-9682-7, 2010.

Doppler data processing for orbit determination

Sander Goossens

Abstract

This brief report describes some basics about Doppler data processing for the purpose of orbit determination. The goal is to write down the formulation for the specific 4-way link for the SELENE measurement. This can then be used when loading the data into GEODYN. The start is a recap of a 2-way data link, which can serve as the basis for the 4-way link.

1 Two-way Doppler

In a non-relativistic formulation, the observed frequency is expressed as:

$$f_{\text{obs}} = \left(1 - \frac{v}{c}\right) f_{\text{sent}} \tag{1}$$

with f_{obs} the observed frequency, f_{sent} the transmitted frequency at the station, v the line-of-sight velocity, and c the speed of light. In a relativistic formulation, the factor $1/\gamma$ needs to be added, with $\gamma = \sqrt{1-(v/c)^2}$. It is assumed here that $v \ll c$.

For a two-way link, there is usually a turn-around ratio T at the satellite to prevent interference from the up- and downlinks. This ratio is included here. In order to construct the received frequency f_{r} as a function from the transmitted frequency f_{t} at the station, the signal is traced back. Firstly, the received frequency depends on the frequency as transmitted by the satellite, $f_{\text{sent, sat}}$, according to equation (1):

$$f_{\text{r}} = \left(1 - \frac{v_2}{c}\right) f_{\text{sent, sat}} \tag{2}$$

with v_2 the line-of-sight velocity for the downlink. However, the frequency of the signal sent at the satellite, equals the observed frequency at the satellite, times the turn-around ratio T:

$$f_{\text{sent, sat}} = T f_{\text{obs, sat}} \tag{3}$$

The observed satellite frequency itself is Doppler shifted, according to

$$f_{\text{obs, sat}} = \left(1 - \frac{v_1}{c}\right) f_{\text{t}} \tag{4}$$

with v_1 the line-of-sight velocity for the uplink, and f_{t} the transmitted frequency at the station. Combining all these equations together yields:

$$f_{\text{r}} = \left(1 - \frac{v_2}{c}\right) T \left(1 - \frac{v_1}{c}\right) f_{\text{t}} \tag{5}$$

This can be simplified, when it is taken into account that $v_1 = v_2 = v$ (the delay at the spacecraft is neglected here, and in any case, it is assumed that the signals are re-transmitted back to the station almost instantaneously), and that quadratic terms $(v/c)^2$ can be neglected, since it is assumed that $v \ll c$ (non-relativistic formulation). This then leads to a well-known formula:

$$f_{\text{r}} = \left(1 - 2\frac{v}{c}\right) T f_{\text{t}} \tag{6}$$

In general, the received frequency is not measured directly, but instead, zero-crossings of the received frequency compared with a reference frequency f_{ref} are. These are integrated along a certain time interval $t_2 - t_1 = T_c$, the count interval. This then leads to a Doppler count, from which the average range-rate can be determined. This is used in GEODYN as an observable. The Doppler count N is defined as (*e.g. Montenbruck and Gill* [2000]):

$$N = \int_{t_1}^{t_2} (f_{\text{r}} - f_{\text{ref}})\, dt \tag{7}$$

Using equation (6) for the received frequency, remembering that $t_2 - t_1 = T_c$, and assuming a constant transmitter frequency and reference frequency, this can be written as

$$N = \int_{t_1}^{t_2} (1 - 2\frac{v(t)}{c}) T f_{\text{t}}\, dt - T_c f_{\text{ref}} \tag{8}$$

The average range rate is defined as $\bar{v} = (1/T_c) \int_{t_1}^{t_2} v(t) dt$, so the above equation can also be written as:

$$N = T f_{\text{t}} \left\{ T_c - 2T_c \frac{\bar{v}}{c} \right\} - T_c f_{\text{ref}} \tag{9}$$

In the case that $f_{\text{ref}} = T f_{\text{t}}$, this can be simplified further, and the constant term $T f_{\text{t}} T_c - T_c f_{\text{ref}}$ drops out. This then finally yields for the average range-rate $\bar{v}$:

$$\bar{v} = -\frac{1}{2} \frac{cN}{T f_{\text{t}} T_c} \tag{10}$$

which equals equation (6.9) of *Montenbruck and Gill* [2000].

2 Four-way Doppler

In the case of SELENE, there is an extra link between two satellites, the main satellite and the relay satellite. This complicates the measurement link somewhat, as there are also different turn-around ratios in the satellites. The final downlink is also a mix of a received signal from the relay satellite and the station uplink. The measurement scenario is first described, and then the measurement link is worked out, following the 2-way Doppler approach.

2.1 Measurement link

In the SELENE scenario, a transmitter frequency is sent to the relay satellite. This is an S-band signal. This signal is then forwarded to the main satellite, after a multiplication using the factor $K_5 = \frac{238}{221}$. This signal is then re-transmitted back from the main satellite to the relay satellite, with a multiplication factor of $K_6 = \frac{270}{295}$. This signal is received in the relay satellite, and mixed with $K2$ times the original received frequency, with $K_2 = \frac{680}{221}$. This results in the last link from the 4-way measurement being in the X-band instead of in the S-band. Apart from that, a normal 2-way link is also included, with the standard turn-around ratio of $K = \frac{240}{221}$ for S-band signals.

2.2 Modelling the 4-way observable

The signal is again traced back as was done for the 2-way model. For each leg, a velocity will be defined, according to figure 1.

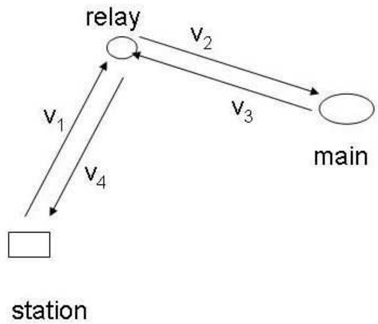

Figure 1. The SELENE 4-way measurement links and velocities definitions

The station transmits again at a frequency f_{t}. In the relay satellite (the tracking satellite), this link is received at a Doppler shifted frequency f_3:

$$f_3 = \left(1 - \frac{v_1}{c}\right) f_{\text{t}} \tag{11}$$

with v_1 the velocity of the the relay satellite with respect to the station on the uplink.

Now working backwards along the signal path, the received frequency at the station f_{r} is received at a Doppler shifted frequency, depending on the signal as broadcast finally from the relay satellite:

$$f_{\text{r}} = \left(1 - \frac{v_4}{c}\right) f_{\text{sent, relay-to-station}} \tag{12}$$

with v_4 the velocity of the relay satellite with respect to the station on the final leg. Thus, v_1 and v_4 constitute a two-way link, as they don't concern the tracking of the main satellite. For now, all velocities are written separately, and it is not necessarily assumed that $v_1 = v_4$ (as was done in the two-way case), as there is some time in between these velocities, namely, the time it takes the signal to go from the relay satellite to the main satellite and back.

According to the lay-out of the SELENE measurement, on-board the relay satellite, the incoming signal from the main satellite (the tracked satellite) is mixed with f_3, and then it is sent to the station as $f_{\text{sent, relay-to-station}}$:

$$f_{\text{sent, relay-to-station}} = K_2 f_3 + f_{\text{received, relay-from-main}} \tag{13}$$

With the expression for f_3 as given in equation (11), this becomes

$$f_{\text{sent, relay-to-station}} = K_2 \left(1 - \frac{v_1}{c}\right) f_{\text{t}} + f_{\text{received, relay-from-main}} \tag{14}$$

Working backwards again along the signal path, the frequency $f_{\text{received, relay-from-main}}$ arrives, Doppler shifted, from the main satellite, and can be expressed as:

$$f_{\text{received, relay-from-main}} = \left(1 - \frac{v_3}{c}\right) f_{\text{main, sent}} \tag{15}$$

with v_3 the velocity between the main and the relay satellite, on the downlink from the main satellite to the relay satellite.

For the main satellite, there is a turn-around ratio K_6 such that

$$f_{\text{main, sent}} = K_6 f_{\text{main, received}} \tag{16}$$

The received frequency at the main satellite finally is again Doppler shifted and comes from the relay satellite:

$$f_{\text{main, received}} = \left(1 - \frac{v_2}{c}\right) f_{\text{relay, sent-to-main}} \tag{17}$$

with v_2 the velocity between main and relay satellite on the uplink between them. The signal as sent by the relay satellite to the main satellite, finally, is given according to

$$f_{\text{relay, sent-to-main}} = K_5 f_3 = K_5 \left(1 - \frac{v_1}{c}\right) f_{\text{t}} \tag{18}$$

Now, all frequencies can be traced back to the initial, transmitted frequency, by subsequently collecting the above equations, and back-substituting them. This then yields for the received frequency:

$$f_{\text{r}} = \left(1 - \frac{v_4}{c}\right) \left\{ K_2 \left(1 - \frac{v_1}{c}\right) f_{\text{t}} + \left(1 - \frac{v_3}{c}\right) K_6 \left(1 - \frac{v_2}{c}\right) K_5 \left(1 - \frac{v_1}{c}\right) f_{\text{t}} \right\} \tag{19}$$

The above equation can be expanded, and quadratic and higher order terms of v/c are neglected, due to the assumption $v \ll c$. This then yields

$$f_{\text{r}} = \left\{ K_2 \left(1 - \frac{v_1}{c} - \frac{v_4}{c}\right) + K_5 K_6 \left(1 - \frac{v_1}{c} - \frac{v_2}{c} - \frac{v_3}{c} - \frac{v_4}{c}\right) \right\} f_{\text{t}} \tag{20}$$

Equation (20) is ready to be used in the formula for the Doppler count N as given in the section dealing with the two-way data, equation (7). The reference frequency is briefly left out of the discussion now, so that the focus will be on the integral $\int_{t_1}^{t_2} f_{\text{r}}\, dt$, with again $t_2 - t_1 = T_c$, the count interval. Integrals of velocities give an average range rate according to

$$\bar{v}_i = \frac{1}{T_c} \int_{t_1}^{t_2} v_i(t)\, dt \tag{21}$$

For now, they are not grouped together, as was done for the two-way signal. Instead, each leg is left apart, so that the final result can be compared directly with the way that the 4-way observable is defined in GEODYN.

Expanding the integral over the received frequency f_{r}, and taking into account the definition of the average range-rates for the separate legs according to equation (21), the integral becomes:

$$\int_{t_1}^{t_2} f_{\text{r}}\, dt = (K_2 + K_5 K_6) f_{\text{t}} T_c - K_2 T_c f_{\text{t}} \frac{\bar{v}_1}{c} - K_2 T_c f_{\text{t}} \frac{\bar{v}_4}{c} - K_5 K_6 T_c f_{\text{t}} \left(\frac{\bar{v}_1}{c} + \frac{\bar{v}_2}{c} + \frac{\bar{v}_3}{c} + \frac{\bar{v}_4}{c} \right) \tag{22}$$

This equation can be cleaned up for now, if it is assumed that the reference frequency against which the received frequency is beat, equals $(K_2 + K_5 K_6) f_{\text{t}}$, similar as in the two-way case. In this case, the constants drop out of the equation again. This isn't completely necessary; they can be maintained, but a bias would be added (or subtracted) from the Doppler count variable N. For brevity and clarity, this is assumed for now.

This then yields the following formula:

$$(\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4) + \frac{K_2}{K_5 K_6} (\bar{v}_1 + \bar{v}_4) = -\frac{1}{K_5 K_6} \frac{Nc}{T_c f_{\text{t}}} \tag{23}$$

This is the observation equation for the 4-way Doppler observable, expressed in the count interval T_c, the transmitted frequency f_{t} and the several turn-around ratios. Namely, according to the GEODYN manual, the 4-way satellite-to-satellite relay Doppler measurement is expressed as being computed along the lines of ([*Rowlands et al.*, 1995])

$$\text{T3} \rightarrow \text{S3} \rightarrow \text{S1} \rightarrow \text{S2} \rightarrow \text{T2} + \text{SF} \times (\text{T2} \rightarrow \text{S2} \rightarrow \text{T2})$$

In the case of SELENE, S3 = S2 and T3 = T2, and that means that the above measurement link matches the sequence as given in equation (23). This also shows readily which value to use for the scale factor SF:

$$\text{SF} = \frac{K_2}{K_5 K_6} \tag{24}$$

In the case of SELENE, using the values for K_2, K_5 and K_6, the scale factor becomes $SF = \frac{680 \cdot 295}{238 \cdot 270} \approx 3.1217$.

Equations (23) and (24) can thus be readily used when pre-processing the data, making them suitable for importing into GEODYN. These equations can also be used when transforming frequency values to metric values, for the use in GEODYN.

References

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